DEPARTMENT: <u>Mathematics</u>   COURSE <u>Advanced Calculus</u>

Curriculum Development Timeline

School: Ocean Township High School

Course: Calculus Advanced

Department: Mathematics

Board Approval	Supervisor	Notes
June 2015	Janet Bluefield	Born Date
August 2017	Nichole Kerney	Revisions
August 2019	Nichole Kerney	Review
August 2022	Gerard Marrone	Alignment to Standards

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	Township of Ocean Pacing Guide		
Week	Marking Period 1	**Week**	Marking Period 3
1	Functions and Equations	11	Applications of Differentiation
2	Functions and Equations	12	Applications of Differentiation
3	Functions and Equations	13	Applications of Differentiation
4	Limits and Continuity	14	Integrals
5	Limits and Continuity	15	Integrals
Week	Marking Period 2	**Week**	Marking Period 4
6	Limits and Continuity	16	Integrals
7	Differentiation	17	Integrals
8	Differentiation	18	Applications of Integration
9	Differentiation	19	Applications of Integration
10	Differentiation	20	Applications of Integration

Core Instructional & Supplemental Materials including various levels of Texts
Calculus AP Edition, 3rd Edition, Pearson Prentice Hall Supplemental: IXL Math

Time Frame	3 Weeks (15 blocks)
Topic	
Functions & Equations	
Alignment to Standards	
F-IF.1, F-IF.2, F-IF.4-9, F-BF.1, F-BF.3-5, F-LE.1, F-LE.5, F-TF.5	

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Learning Objectives and Activities

SWBAT answer the following questions:
- Can the student demonstrate knowledge of the following types of functions: linear, power, polynomial, piecewise, exponential, logarithmic, logistic, rational and trigonometric?
- Can the student represent patterns and relationships graphically, numerically, symbolically and/or verbally?
- How can patterns, relations and functions be used to describe real-life situations?

SWBAT demonstrate understanding of the following:
- Mathematical modeling is a process to construct a mathematical framework to represent real world situations.
- Patterns and relationships can be represented graphically, numerically, symbolically and/or verbally.

Learning Activities:
- Graph linear using a point and the slope
- Determine a linear equation of using a point and the slope
- Graph polynomials using roots, minima, maxima, y-intercept, end behavior model, multiplicity, domain/range, intervals of increase and decrease
- Determine a polynomial equation of using roots, multiplicity and end behavior model
- Simplify expressions using algebraic techniques
- Evaluate and graph piecewise functions.
- Determine a piecewise equation of given graph
- Determine if the piecewise function is continuous
- For rational functions determine points of discontinuity, x-intercept(s), y-intercept, hole(s), vertical asymptote(s), horizontal asymptote and oblique asymptote
- Graph rational functions using its characteristics
- For exponential, logarithmic, and logistic functions determine the asymptotes, domain, range, x-intercept(s) and y-intercept
- Graph and evaluate exponential, logarithmic, and logistic functions using its characteristics
- Evaluate expressions involving trigonometric and inverse trigonometric functions
- Graph sinusoids
- Solve problems involving right triangle trigonometry
- Graph implicitly defined functions

Assessments

<u>Formative:</u>
- Anticipatory set/exit ticket
- Daily Practice Problems

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- Teacher Observation
- Graphic Organizer

<u>Summative:</u>
- Mid-unit Quizzes
- Topic Tests

<u>Alternative:</u>
- Project involving graphing all functions
- PollEverywhere.com
- Individual or group productive struggle assessment during introductory lessons

Interdisciplinary Connections

Science: HS-ETS1-2: In this functions unit students will design a solution to a complex real-world problem involving maximum/minimum/projectile motion by breaking it down into smaller, more manageable problems that can be solved through engineering.

Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.6: In the logarithms unit students will describe and calculate interest and fees that are applied to various forms of spending, debt, and saving in solving equations.

Technology Integration

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.
- 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.

Students will use internet based game sites such as Quizizz, Kahoot, and Quizlet live to reflect on their learning progress.
- 9.4.5.CT.3: Describe how digital tools and technology may be used to solve problems.

Students will use Google Classroom to collaborate, work towards solving authentic problems, or participate in an online classroom discussion utilizing pre-learned etiquette about blended learning platforms.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

Students will use graphing calculators to use math tools strategically and attend to precision and will use Desmos in order to discover new concepts involving graphing and functions.

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- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping

Career Education
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them. CRP11: Use technology to enhance productivity.

Time Frame	3 Weeks (15 blocks)
Topic	
Limits and Contunity	
Alignment to Standards	
F-IF.1, F-IF.2, F-IF.4-9, F-BF.3-5, F-LE.1, F-LE.5, F-TF.5	
Learning Objectives and Activities	

SWBAT answer the following questions:
- What is the limit of a function?
- How can we evaluate limits numerically, graphically and analytically?
- How can limits be used to analyze functions?
- What are the behaviors that would cause nonexistence of a limit?
- How is the derivative defined and what does this mean geometrically?
- How are average and instantaneous rates of change related?
- How do we find the equation of the tangent line?

SWBAT demonstrate understanding of the following:
- Limits can be used to analyze functions numerically, graphically and analytically.
- Functions can behave differently at different points in their domain.
- A function's continuity can be determined using limits.
- Tangent lines to curves determine an instantaneous rate of change

Learning Activities:
Finding limits
- Find one-sided and two-sided limits graphically
- Evaluate limits of polynomial, rational and trigonometric functions

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- Evaluate limits of functions involving absolute value and piecewise functions
- Find one-sided and two-sided limits involving infinity
- Evaluate limits of rational (using asymptotes), polynomial (using end behavior model), exponential and trigonometric functions
- Evaluate limits of piecewise, rational (by factoring, simplifying or using the conjugate) and trigonometric functions algebraically

Continuity
- Define continuity formally and informally
- Identify the different types of discontinuities (infinite, removable, jump and oscillating)
- Find the intervals on which a function is continuous
- Apply the Intermediate Value Theorem

Average Rate of Change
- Calculate the average rate of change of a function over the given interval
- Find the equation of the secant line

Definition of a Derivative
- Use the definition of the derivative to find the derivative of a function (linear, quadratic, rational and square root) with respect to x
- Calculate the instantaneous rate of change at a point of a function
- Find the equation of the tangent and normal lines at a point of a function

Assessments

<u>Formative:</u>
- Classwork and Homework
- Teacher Observation
- Class Debate of Approaches/Mathematical Methods
- Math Scavenger Hunt/Trail

<u>Summative:</u>
- Mid-unit Quizzes
- Topic Tests
- Problem-based Quiz/Test on Application

<u>Alternative:</u>
- Exploration – unique challenges to study concepts as reinforcement and/or study concepts not yet formally covered
- Observation Assessment with Problem-solving
- Kahoot/Quizizz

Interdisciplinary Connections

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Career Readiness, Life Literacies, and Key Skills

9.1.12.CDM.8: When learning exponential equations students will compare and compute interest and compound interest.

Technology Integration

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Students will use graphing calculators to use math tools strategically and attend to precision and will use Desmos in order to discover new concepts involving graphing and functions.
- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping

Career Education

CRP6: Demonstrate creativity and innovation.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.

Time Frame	4 Weeks (20 blocks)
Topic	
Differentiation	

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Alignment to Standards
F-IF.1, F-IF.2, F-IF.4-9, F-BF.3-5, F-LE.1, F-LE.3, F-LE.5, F-TF.5, F-TF.6

Learning Objectives and Activities

SWBAT answer the following questions:
- How does the derivative relate to the concept of change?
- How can the rate of change of a function help develop the graph?
- What does a derivative represent?
- How can the derivative be found using analytic methods?
- What are the derivative rules for functions?

SWBAT demonstrate understanding of the following:
- The relationship between a function and its derivative can be explored utilizing various methods (analytical, graphical and numerical).

Learning Activities:
Differentiability
- Determine if a function is differentiable at a value of x and determine the reason why it might fail to exist at a value of x (discontinuities, vertical tangents, corners and cusps)
- Differentiability implies continuity, but continuity does not imply differentiability
- Graph the derivative given the graph of the original function and vice versa

Power, Product, Quotient and Chain Rule
- Calculate the derivative of polynomial functions and functions with radicals using the power rule
- Write the first derivative using proper notation
- Calculate the derivative of a function using the product, quotient and chain rule

Higher Order Derivatives
- Calculate the second, third and fourth derivative of a polynomial function using the power rule
- Write the second, third and fourth derivative using proper notation

Trigonometric, Exponential, and Logarithmic Functions
- Calculate the derivative of the six basic trigonometric and six inverse trigonometric functions
- Calculate the derivative of exponential and logarithmic functions with base e and base not equal to e

Implicit Differentiation
- Calculate the derivative of equations that are described by complicated equations that are difficult or impossible to solve for y

Assessments

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Formative:
- Classwork and Homework
- Teacher Observation
- Class Debate of Approaches/Mathematical Methods
- Anticipatory set/exit ticket

Summative:
- Mid-unit Quizzes
- Topic Tests

Alternative:
- PollEverywhere.com
- Observation Assessment with Problem-solving
- Kahoot/Quizizz
- Individual or group productive struggle assessment during introductory lessons

Interdisciplinary Connections

Career Readiness, Life Literacies, and Key Skills

Technology Integration

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.
- 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.

Students will use internet based game sites such as Quizizz, Kahoot, and Quizlet live to reflect on their learning progress.
- 9.4.5.CT.3: Describe how digital tools and technology may be used to solve problems.

Students will use Google Classroom to collaborate, work towards solving authentic problems, or participate in an online classroom discussion utilizing pre-learned etiquette about blended learning platforms.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

Students will use graphing calculators to use math tools strategically and attend to precision and will use Desmos in order to discover new concepts involving graphing and functions.

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- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping

Career Education
CRP6: Demonstrate creativity and innovation. CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.

Time Frame	3 Weeks (15 blocks)
Topic	
Application of Differentiation	
Alignment to Standards	
[F-IF.1, F-IF.2, F-IF.4-9, F-BF.3-5, F-LE.1, F-LE.3, F-LE.5, F-TF.5, F-TF.6](#)	
Learning Objectives and Activities	

SWBAT answer the following questions:
- How can derivatives help us solve real life application problems?
- What does the first and second derivative tell us about a function?
- What does the concavity of a curve determine?
- What do horizontal tangent lines represent?
- How can functions and their derivatives be used to minimize or maximize situations?

SWBAT demonstrate understanding of the following:
- Derivatives are used to solve real life problems involving maximizing or minimizing quantities in both physical and social sciences.

Learning Activities:
Curve Sketching
- Find the absolute and local extrema using derivatives
- Apply the Extreme Value Theorem
- Apply the Mean Value Theorem
- Apply Rolle's Theorem
- Apply the First Derivative Test for Local Extrema
- Apply the Second Derivative Test for Local Extrema

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- Determine when a function is increasing or decreasing using the derivative
- Find critical points
- Find points of inflection
- Find the concavity of a function given a value of x

Optimization
- Find the minimum or maximum by finding a function to model a situation and then calculate the derivative

Related Rates
- Use derivatives to solve problems that involve objects in motion

Assessments

Formative:
- Teacher Observation
- Class Debate of Approaches/Mathematical Methods
- Math Scavenger Hunt/Trail
- Entrance/Exit Cards

Summative:
- Mid-unit Quizzes
- Topic Tests
- Problem-based Quiz/Test on Application

Alternative:
- PollEverywhere.com
- Observation Assessment with Problem-solving
- Individual or group productive struggle assessment during introductory lessons

Interdisciplinary Connections

Career Readiness, Life Literacies, and Key Skills

Technology Integration

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.
- 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem.

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Students will use internet based game sites such as Quizizz, Kahoot, and Quizlet live to reflect on their learning progress.
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- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

Students will use graphing calculators to use math tools strategically and attend to precision and will use Desmos in order to discover new concepts involving graphing and functions.
- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping

Career Education
CRP2: Apply appropriate academic and technical skills. CRP6: Demonstrate creativity and innovation.

Time Frame	4 Weeks (20 blocks)
Topic	
Integrals	
Alignment to Standards	
F-IF.1, F-IF.2, F-IF.4-9, F-BF.3-5, F-LE.1, F-LE.5, F-TF.5	
Learning Objectives and Activities	

SWBAT answer the following questions:
- How can you find the area of a bounded region using integration?
- How do we find the antiderivative analytically?
- What are the different methods for integration?
- What is the Fundamental Theorem of Calculus?

SWBAT demonstrate understanding of the following:

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- There are different methods of integration.
- Distance can be visualized as area under the curve.
- Derivatives and integrals are related.

Learning Activities:
Indefinite Integration
- Calculate the integral of polynomials and expressions involving radicals by applying the power rule
- Calculate the integral of logarithmic and exponential functions
- Calculate the integral of trigonometric functions
- Calculate the integral of inverse trigonometric functions

Methods of Integration
- Calculate the integral of functions (power rule, logarithmic, exponential, trigonometric and inverse trigonometric) by using substitution
- Calculate the integral of functions by using integration by parts

Definite Integration
- Calculate the approximate area under a curve using LRAM, MRAM, RRAM and the Trapezoidal Rule
- Calculate the value of a definite integral using The Fundamental Theorem of Calculus and the graph of the function
- Discover the properties of definite integrals by analyzing the area under a curve
- Calculate the value of a definite integral using substitution with change of variables
- Apply the Mean Value of Definite Integrals

Assessments

Formative:
- Classwork and Homework
- Class Debate of Approaches/Mathematical Methods
- Graphic Organizer
- Entrance/Exit Cards

Summative:
- Mid-unit Quizzes
- Topic Tests

Alternative:
- Observation Assessment with Problem-solving
- Kahoot/Quizizz
- Individual or group productive struggle assessment during introductory lessons

Interdisciplinary Connections

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Career Readiness, Life Literacies, and Key Skills

Technology Integration
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Career Education
CRP4: Communicate clearly and effectively with reason. CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.

Time Frame	3 Weeks (15 blocks)
Topic	
Applications of Integration	
Alignment to Standards	

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<u>F-IF.1, F-IF.2, F-IF.4-9, F-BF.3-5, F-LE.1, F-LE.5, F-TF.5</u>

Learning Objectives and Activities

SWBAT answer the following questions:
- How can we find the area between curves using integration?
- How can we find the volume of solids using integration?

SWBAT demonstrate understanding of the following:
- Integrals can be used to find the area between curves.
- Integrals can be used to find the volume of solids.

Learning Activities:
Area
- Calculate the area under a curve
- Calculate the area between curves

Volume
- Calculate the volume of solids by slicing (disks and washers)
- Calculate the volume of solids by cylindrical shells
- Calculate the volume of solids with known cross sections

Assessments

Formative:
- Classwork and Homework
- Daily Practice Problems
- Teacher Observation
- Class Debate of Approaches/Mathematical Methods
- Anticipatory set/exit ticket

Summative:
- Mid-unit Quizzes
- Topic Tests
- Problem-based Quiz/Test on Application

Benchmark:
- Cumulative final exam with multiple choice, short answer, and extended constructed response questions.

Alternative:
- Project – Integrals in the real-world
- Kahoot/Quizizz
- Individual or group productive struggle assessment during introductory lessons

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Interdisciplinary Connections

ELA: W.11-12.1: When students are justifying their reasoning on short answer and extended constructed response questions they write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

Career Readiness, Life Literacies, and Key Skills

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Career Education

CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11: Use technology to enhance productivity.

Modifications (ELL, Special Education, At Risk Students, Gifted & Talented, & 504 Plans)

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ELL:
- Work toward longer passages as skills in English increase
- Use visuals
- Introduce key vocabulary before lesson
- Teacher models reading aloud daily
- Provide peer tutoring
- Use of Bilingual Dictionary
- Guided notes and/or scaffold outline for written assignments
- Provide students with English Learner leveled readers.

Supports for Students With IEPs:
- Allow extra time to complete assignments or tests
- Guided notes and/or scaffold outline for written assignments
- Work in a small group
- Allow answers to be given orally or dictated
- Use large print books, Braille, or books on CD (digital text)
- Follow all IEP modifications

At-Risk Students:
- Guided notes and/or scaffold outline for written assignments
- Introduce key vocabulary before lesson
- Work in a small group
- Lesson taught again using a differentiated approach
- Allow answers to be given orally or dictated
- Use visuals / Anchor Charts
- Leveled texts according to ability

Gifted and Talented:
- Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles)
- Provide options, alternatives and choices to differentiate and broaden the curriculum
- Organize and offer flexible small group learning activities
- Provide whole group enrichment explorations
- Teach cognitive and methodological skills
- Use center, stations, or contracts
- Organize integrated problem-solving simulations
- Propose interest-based extension activities

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- Expose students to beyond level texts.

Supports for Students With 504 Plans:
- Follow all the 504 plan modifications
- Text to speech/audio recorded selections
- Amplification system as needed
- Leveled texts according to ability
- Fine motor skill stations embedded in rotation as needed
- Modified or constrained spelling word lists
- Provide anchor charts with high frequency words and phonemic patterns